presentation of an optimal design procedure using an NLP algorithm should, therefore, address the reliability question if the potential user of the procedure is to be properly informed as to its performance.

The recent article by Bronowicki et al.[2] demonstrates the need for optimality confirmation and a discussion of solution reliability. The problems treated there are quite difficult in the NLP sense. Application of the DSDA (a procedure which appears reasonably reliable, solving 9 of 10 of the problems of Eason and Fenton)[3], to a problem very similar to the type 1 problem of Ref. 2 produced optimization algorithm failure. It was necessary to resort to variable coupling and mixed discrete-continuous programming techniques to obtain solution reliability.[4] SUMT procedures such as used in Ref. 2 appear to be substantially less reliable than DSDA. The best of the SUMT based procedures tested by Eason and Fenton solved only half of the test problems.[1] Thus, it seems reasonable to suspect reliability problems with such a procedure on this difficult shell problem.

The 1,000 ft study for problem type 1 of Ref. 2 was checked by a new code using constraint equations, which are identical to those of Ref. 2, except for the hull vibration and buckling equations which are quite similar.[5] The shall parameter values of Ref. 2 were used in the study. A value for the weight/displacement ratio was obtained that is about 5% lower than that reported in Ref. 2 and some 19% lower than reported in the original report[6] on which Ref. 2 is based. It should be noted that the design reported in Ref. 2 cannot be considered typical of the performance of the optimization procedure used there since it was developed on the basis of information generated by another similar, shell optimization capability.[2,4] A difference of 5%, although perhaps inconsequential considering the accuracy of the behavior equations used, is significant from a mathematical programming viewpoint. Thus, the design for the 1,000 ft, type 1 problem in Ref. 2 does not appear to be near optimum in a mathematical programming sense.

Furthermore the design presented as optimal in Ref. 6, and as a local optimum in Ref. 2, is apparently neither. This design was used as a starting point for a synthesis run using the program of Ref. 5. The search immediately moved away from this point. Since the minimum natural frequency constraint and shell buckling constraints are not active for this design, and thus the active constraints equations for this point are identical to those of Ref. 2 and 6, one can conclude that the point is not a local optimum. Thus it appears the NLP procedure used in Ref. 2 simply failed at this point.

This writer concludes, on the basis of his studies,[5] earlier comparison studies,[1] the difficulty of the problem,[4] the differing results presented in Refs. 2 and 6, and the starting point sensitivity cited in Ref. 2, that the optimization procedure described therein is unsuitable for the problems posed.

Some additional, less important, points are also noteworthy. Although the limitation on the vibration mode search to the range $0 \leq n \leq 6, 1 \leq m \leq 6$ does not affect any of the results presented, it should be noted that the use of this limitation could produce invalid designs for the parameters used in the 1,000 ft case if a minimum natural frequency of zero is specified since an optimal design with these parameters will have both a boundary ($m=1$) and interior ($m \geq 6$) buckling minimum active.[7]

The elimination of the inconsistency of using a static rather than dynamic constraint for the interring shell mode can be effected without difficulty by applying the frequency equations of Ref. 2 to the interring shell segment. The treatment of modes shapes for the interring shell panel would not be inconsistent with the use of an orthotropic shell model[2] since the panel is unstiffened and thus the model reduces to the isotropic case where the limitations cited in Ref. 2 do not apply.

Considering the form of the objective function and parameters used in the paper, the procedure appears intended for the design of shells submersed in water. Yet at least for some of the paramenters studied, the in-vacuo frequencies such as those employed in Ref. 2 are a poor approximation to the frequencies of immersed shells. Studies of a stiffened shell with R/t_s and L/R ratios very close to those of the 1,000 ft case show that the lowest frequencies in water are about half of those in air.[8] Thus, it seems unrealistic to specify a minimum in-vacuo frequency when the actual frequencies may be much lower. Furthermore it seems inappropriate to employ NLP methods in a situation where one cannot obtain a reasonable estimate of the merit of a design. Thus the applicability of problem types 2 and 3 to submerged shells seems questionalble, since in-vacuo frequencies are used in the objective function of these problems.

Finally the differences between the weights of the designs presented in Ref. 2 and Ref. 4 are not due in any significant degree to variable coupling as stated in Ref. 2. Rather they are due primarily to the inclusion in Ref. 4 of a stiffener buckling constraint,[9] which is apparently unjustified and not used in Ref. 2.

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Reply by Authors to M. Pappas

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PAPPAS comments on two main points regarding the optimization study in Ref. 1, namely that: 1) the numerical

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optimization algorithm is unreliable and therefore unsuitable, and 2) various aspects of the dynamical analysis are not applicable to the study of submerged, ring-stiffened cylinders.

With regard to the first point, studies of the type performed by Eason and Fenton[2] (to which Pappas refers) show that the possibility of algorithm failure is always present, and that "reliable" algorithms are those with an acceptable success/failure ratio when applied to test problems. Pappas has recognized this situation[3] noting that the DSDA algorithm which he used in his studies can fail. He indicates that repeated computer runs using different starting designs will help to resolve this difficulty. Eason and Fenton also note that although they experienced problems with the SUMT-Davidon-Fletcher-Powell algorithm of the type used in Ref. 1, other researchers gave it a high rating.

User preference of one design algorithm over another depends, in addition to its "reliability", upon its efficiency and versatility in application to different design problems. The SUMT-Davidon-Fletcher-Powell algorithm was chosen for use in Ref. 1 because it was general enough to be used in three different types of design problems without having to be "tuned" for any specific case. The 5% difference between minimum-weight designs reported in Ref. 1 and those claimed by Pappas are viewed by the authors as being in close agreement and not an adequate basis for raising questions concerning algorithmic capabilities. It should be noted that in Ref. 1 a Flugge-Lure-Byrne shell theory is used in a dynamic shell buckling analysis, while in Ref. 3 a static analysis is used based on a Donnell theory.

In either study, the primary objective must be to determine the impact that formal design procedures can have on practical design problems. Results of the type in Refs. 1 and 3 can serve as guidelines for designers who must consider, in addition to the well-defined constraints on structural behavior, nonbehavioral complications such as economics, ease of fabrication and nonstructural requirements.

The second point raised by Pappas concerned the suitability of the dynamic analysis used in the design studies. He points out that for certain cases more longitudinal bending modes should be considered in the dynamic buckling analysis. As was noted in Ref. 1, the use of more than approximately six longitudinal half-waves would be in conflict with the "smearing" of the frame stiffnesses along the length of the shell, since the additional modes have buckling wave lengths which are typically less than twice the frame spacing.

Pappas suggests that inter-ring panel stability should be treated in a dynamical analysis by using the shell model in Ref. 1 for each panel. This would be valid only if the panels were simply supported, but clearly, T-ring motion will interact with the panel through interface conditions more complex than the simple support type. Thus, the proposed shell panel model could not accurately represent the system behavior.

Although the authors agree that the influence of immersion is important, the complete fluid-structure interaction analysis is a complex numerical problem which is currently too expensive for inclusion in an optimization algorithm. The natural modes of in vacuo vibrations are an important tool in simplifying this problem[4] and can be used as an initial basis for investigating the degree to which optimization algorithms can change the response characteristics of a structure. Design problems 2 and 3 (Ref. 1) are both exploratory studies of this type, problem 2 dealing with frequency separation and problem 3 dealing with the separation of axial frequencies. For each design problem, important information was obtained: in problem 2 the lowest two frequencies could be separated with the result that higher frequencies became nearly coincident, while in problem 3 axial frequencies could be separated with the result that "axial" and "radial"-type structural motions became more nearly alike.

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Errata

Experimental Investigation of Underexpanded Exhaust Plumes

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THE upper bound of the integral appearing in Eq. (2) should be θ_∞ instead of θ_0. Thus Eq. (2) should read

$$A = R_e^2 \left[\frac{1}{1+\cos\alpha_e} - \frac{\delta_1}{R_e} + \left(\frac{\delta_1}{R_e}\right)^2 \frac{\cos\alpha_e}{2} \right]$$

$$\Big/ \int_0^{\theta_\infty} \left[\cos\left(\frac{\pi}{2}\frac{\theta}{\theta_\infty}\right)\right]^{2/(\gamma-1)} \sin\theta \, d\theta$$

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